

XLV. A second Letter to the Right Hon. the Earl of Macclesfield, President of the Royal Society, concerning the Transit of Venus over the Sun, on the 6th of June 1761; by the Rev. Nathanael Blifs, M. A. Savilian Professor of Geometry in the University of Oxford, and F. R. S.

My Lord,

Read Jan 7,
1762.

THE interior conjunctions of the planets Mercury and Venus, that happen near the ecliptic limits, have always engaged the attention of astronomers, as they furnish the best means of determining some of the most important elements in the theory of those Planets. The transits of the former have been often and carefully observed by the most eminent astronomers, ever since the invention of the telescope; and, it may be presumed, that the elements of Mercury's theory are established as accurately as can be expected. The opportunities of observing Venus upon the sun's disk occur so seldom, that the astronomers of these days have reason to think themselves peculiarly happy, in being eye-witnesses of so rare a phenomenon; more particularly too, as the advantages resulting from the observations of this transit, are, in all probability, of the greatest moment. The first, and only observation of this kind, was made by our ingenious countryman, the Rev. Mr. Jeremiah Horrox, a young gentleman of very distinguished abilities, who, by his own observations, with instruments

ments constructed under his own inspection, and finished by his own hands, was enabled to correct the so much boasted tables of Lansbergius, and to predict, with a degree of precision unknown to those times, a phenomenon, which he himself thought to be of great consequence. He immediately communicated this important discovery to his friend, and companion in his astronomical studies, Mr. William Crabtree, and earnestly exhorted him to prepare for the observation. The state of the heavens, on that day, was not very favourable: however, both Mr. Horrox and his friend were lucky enough to observe it; the former, at a time when the limbs of the sun and Venus were in the point of contact, viz. on the 24th of November 1639, O. S. And these two were the first, and only persons, that ever saw Venus in the sun, before the present year.

By the Rudolphine tables, constructed from the observations of Tycho Brahe, Kepler was enabled to predict, in the year 1629, that Venus would pass over the sun's disk in the year 1761: and my worthy predecessor, that eminent astronomer and mathematician, Dr. Halley, in a memoir published in the Philosophical Transactions, N^o 348. exhorted the astronomers of all countries to attend to this rare phenomenon, with all possible diligence; as it would furnish them with the best means of determining the parallax and distance of the sun, and, consequently, the dimensions of the whole solar system. How far the method proposed by him, will enable us to solve this difficult problem, must be left to time to discover, when the observations, made in places properly situated, can be compared with those made here, and

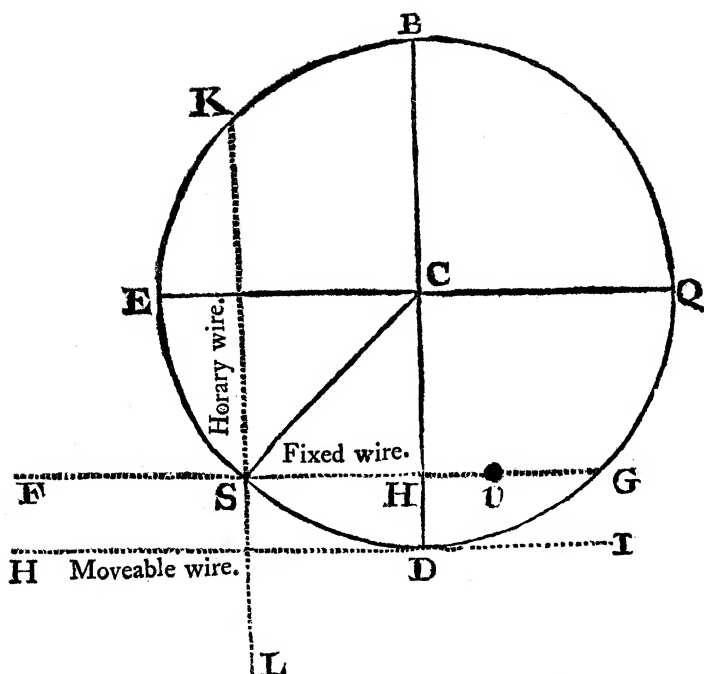
in other famous observatories. The attention paid to the opinion of an English astronomer, by the most renowned Princes, more particularly by his late Majesty, at the request of your Lordship, and the Royal Society, will reflect the greatest honour upon their names, to the latest posterity. But as the tables, which Dr. Halley made use of, were very imperfect, his own not being then constructed, and did not represent the place of Venus on the sun with that accuracy, which the method, in this case, required: and as that eminent philosopher committed a small mistake in his calculations, by placing the axis of Venus's path, and the axis of the equator, on the same side of the axis of the ecliptic; a mistake which the most accurate calculator might easily fall into: from these considerations, I say, the honour of determining the sun's true parallax is, probably, reserved for the reign of his present Majesty; from whom, as a patron of science, and every useful art, we have the greatest reason to promise ourselves every possible encouragement and assistance.

I have already had the honour of presenting to your Lordship, and the Royal Society, an account of the observations of the contacts of the sun's and Venus's limbs, made at Greenwich, and at your Lordship's own observatory. As the time would not then permit me to examine the observations made with the micrometer, I could only select a few particulars, relating to the diameters of the sun and Venus, as measured by different observers. I have since had leisure to examine all the observations made upon the day of the transit, both at Shirburn castle, and at the Royal Observatory at Greenwich; and shall now beg
leave

leave to lay before you, both the observations themselves, and the several results deducible from them by calculation.

The method of determining the right ascension and declination of the center of Venus from that of the sun, was the same which Dr. Bradley used, in observing a former transit of Mercury. The planet was made to run down the fixed wire of the micrometer, and the difference of the time of passage was observed between it, and that part of the sun's limb, which was cut by that wire; and the moveable wire was brought to touch the sun's lower limb. If the sun's lower limb had been made to run down the fixed wire, and the moveable wire brought to the planet, and the difference of the time of passage had been observed between it and the sun's consequent limb, on the supposition, that the wire was not exactly parallel to the diurnal motion, it would have caused a considerable error in the difference of right ascension, observed at the distance of the sun's semidiameter. But the method we made use of requires some calculation, to determine the position of Venus on the sun's disk.

FIG. 1.



Let, therefore, in Fig. 1. the circle $EDQB$ represent the sun's disk, in which let EQ be parallel to the equator, and BD an hour-circle; let the pricked line FG represent the fixed wire of the micrometer, HI the moveable wire, and KL the perpendicular or horary wire. The difference of right ascension Hv , and of declination CH , will be determined in the following manner: SC , or CD , the semidiameter of the sun is given, and $CD - DH = CH$, the difference of declination; and SC and CH being given, SH may be found; and then the observed difference of right ascension Sv being diminished

minished in the ratio of radius to the sine of the polar distance of Venus, will give HU, the difference of right ascension.

As the clouds began to disperse, and the sky to become favourable, at Shirburn castle, above two hours before we had any opportunity of observing at Greenwich, I shall first give the observations there made by Mr. Hornsby, and afterwards my own at Greenwich. But here I would beg leave to premise, that, though the numbers are given to parts of a second, the observers do not pretend to an imaginary exactness, (for they did not estimate the times of passage nearer than a quarter of a second of time) but the numbers are such as result from the turning minutes and seconds of time into motion, and the revolutions and parts of the screw of the micrometer into minutes and seconds.

The sun's horizontal diameter, as measured by the micrometer, was $31' 33''$, and that of Venus, by several observers, $58''$; the following observations were therefore deduced, by assuming the semidiameter of the sun $= 15' 46''.5$, and that of Venus $= 29'$.

1. At $17^h 33' 50''$, apparent time, at Shirburn, the center of Venus preceded the part of the sun's limb, cut by the fixed wire, $12' 3\frac{3}{4}''$ in motion; and the north, or upper limb of Venus, was north of the southern, or lower limb of the sun, $6' 29''.6$: therefore, the center of the sun preceded the center of Venus in right ascension $1' 36''.9$; and the center of Venus was south of that of the sun in declination $9' 45''.8$. The same, to avoid repetition, in all the following observations.

2. At

2. At $17^h 35' 41''$, the center of Venus preceded the sun's limb $12' 13''$; and the upper limb of Venus was north of the sun's lower limb $6' 19''.7$: sun's center, therefore, before that of Venus in right ascension, $1' 21''$; and the center of Venus was south of the sun's center in declination $9' 55.7''$.

3. At $17^h 40' 1''$, Venus before sun's limb $12' 37\frac{1}{4}''$, and was north of sun's lower limb $6' 18.2''$: therefore, sun's center, before that of Venus in right ascension, $57''.2$; and Venus south of sun's center in declination $9' 57''.3$.

N. B. In these observations, the sun's limb undulated.

4. At $17^h 43' 59''$, Venus before sun's limb $12' 47''$; and was north of sun's lower limb $6' 16''.3$: therefore, sun's center before Venus in right ascension $47''$; and Venus south of sun's center in declination $9' 59''.2$.

5. At $17^h 50' 31''$, Venus before sun's limb $13' 9\frac{1}{4}''$; and was north of sun's lower limb $6' 10''.4$: therefore, sun's center before Venus in right ascension $22''$; and Venus south of sun's center in declination $10' 5''.1$.

6. At $18^h 3' 41''$, Venus before sun's limb $13' 48\frac{3}{4}''$; and was north of sun's lower limb $5' 59''.2$: therefore, the center of Venus was before the sun's center in right ascension $23''.2$; and was south of sun's center in declination $10' 16''.3$.

7. At $18^h 8' 54''$, Venus before sun's limb $14' 13''$; and was north of sun's lower limb $5' 53''.8$: therefore, Venus before sun's center in right ascension $49''.9$; and was south of sun's center in declination $10' 21''.7$.

8. At

8. At $18^h 15' 50''$, Venus before sun's limb $14' 33''$; and was north of sun's lower limb $5' 48''$: therefore, Venus before sun's center in right ascension $1' 13''.2$; and was south of sun's center in declination $10' 27''.5$

9. At $18^h 28' 6''$, Jenus before sun's limb $15' 9\frac{1}{4}''$; and was north of sun's lower limb $5' 35''.4$: therefore, Venus before sun's center in right ascension $1' 57''.2$; and was south of sun's center in declination $10' 40''.1$.

10. At $19^h 18' 49''$, Venus before sun's limb $18' 5\frac{1}{2}''$; and was north of sun's lower limb $4' 45''.5$: therefore, Venus before sun's center in right ascension $5' 25''.3$; and was south of sun's center $11' 30''$.

11. At $19^h 22' 37''$, Venus before sun's limb $18' 9\frac{1}{4}''$; and was north of sun's lower limb $4' 44''.3$: therefore, Venus before sun's center in right ascension $5' 29''.9$; and was south of sun's center $11' 31''.2$.

12. At $19^h 25' 50''$, Venus before sun's limb $18' 23\frac{3}{4}''$; and was north of sun's lower limb $4' 42''.5$: therefore, Venus before sun's center in right ascension $5' 45''$; and was south of sun's center $11' 32''.9$.

13. At $19^h 29' 20''$, Venus before sun's limb $18' 31\frac{3}{4}''$; and was north of sun's lower limb $4' 35''.2$: therefore, Venus before sun's center in right ascension $5' 59''.8$; and was south of sun's center in declination $11' 40''.3$.

14. At $19^h 45' 58''$, Venus before sun's limb $19' 20\frac{1}{4}''$; and was north of sun's lower limb $4' 19''$: therefore, Venus before sun's center in right ascension $7' 1''.5$; and was south of sun's center $11' 56''.5$.

15. At $19^h 49'$, Venus before sun's limb $19' 28''$; and was north of sun's lower limb $4' 16''.6$: therefore,

fore, Venus before sun's center in right ascension $7^{\circ} 11''.1$; and was south of sun's center $11^{\circ} 58''.9$.

16. At $20^h 12' 1''$, the center of Venus followed the sun's preceding limb, cut by the fixed wire, $1^{\circ} 58''$; and was north of sun's lower limb $3^{\circ} 54''.2$: therefore, Venus before sun's center in right ascension $8^{\circ} 34''.2$; and was south of sun's center in declination $12^{\circ} 21''.3$.

The following observations were made by myself, at Greenwich, as soon as the sky became favourable.

1. At $19^h 38' 21''$, apparent time, at Greenwich, the antecedent, or first limb of Venus, preceded that part of the sun's limb cut by the fixed wire $18^{\circ} 48\frac{1}{2}''$ in motion; and the center of Venus was north of the southern, or lower limb of the sun, $4^{\circ} 4''.5$: therefore, the center of Venus preceded the sun's center in right ascension $6^{\circ} 18''.9$; and was south of that of the sun in declination $11^{\circ} 42''.1$.

2. At $19^h 42' 9''$, the limb of Venus before sun's limb $18^{\circ} 52\frac{1}{2}''$; and was north of sun's lower limb $3^{\circ} 56''.8$: therefore, Venus before sun's center in right ascension $6^{\circ} 31''$; and was south of it in declination $11^{\circ} 49''.7$. But this is marked as dubious.

3. At $19^h 44' 35''$, limb of Venus before sun's limb $19'$; and center was north of sun's lower limb $3^{\circ} 57''.5$: therefore, Venus before sun's center in right ascension $6^{\circ} 37''.1$; and was south of sun's center in declination $11^{\circ} 49''$.

4. At $19^h 53' 14''$, limb of Venus before sun's limb $19^{\circ} 22\frac{1}{2}''$; and was north of sun's lower limb $3^{\circ} 51''.3$: therefore, Venus before sun's center in right ascension $7^{\circ} 5''$; and was south of sun's center in declination $11^{\circ} 55''.2$.

5. At

5. At $19^h 58' 26''$, limb of Venus before sun's limb $19' 37\frac{1}{2}''$; and was south of sun's lower limb $3' 43''$: therefore, Venus before sun's center in right ascension $7^h 28''.6$; and was south of sun's center in declination $12' 3''.5$.

The few observations, which were afterwards made by Mr. Green, with Mr. Dollond's micrometer, are omitted; for they disagree so much with themselves, and also with the above, that there must be some error in reading the numbers of the nonius; or, which is more probable, in placing the micrometer exactly parallel to the equator, occasioned by the hurry with which they were made.

In order to determine more exactly the time of the ecliptic conjunction, with the latitude of Venus then; together with the time of the middle of the transit, and the nearest approach of the centers; and from thence the true place of her node; I have carefully computed the following numbers from theory: because, as Dr. Halley has observed, in the Philosophical Transactions, N^o 386, "there is always an unavoidable, though small uncertainty in what we observe, yet greater than there can be in the theory, especially now it is so very near the truth." The solar numbers were computed from new tables, not yet published, corrected by the small equations, occasioned by the influence of the moon and planet Jupiter, and also the nutation of the earth's axis. The sun's place was very well observed on the meridian, both at Greenwich and Shirburn, the day of the transit; which, allowing for the difference of longitude of those places, agreed to a surprizing exactness, within two seconds; and did not differ more than five

seconds in excess from the computed place. The place of Venus was computed from Dr. Halley's tables, only adding $31''$ to the mean motion, and $1' 45''$ to the place of the node; by which corrections, they had been found to agree better with observations made near the inferior conjunction in 1753.

According to these numbers, the ecliptic conjunction of the sun and Venus was June 5, 1761, N. S. at $17^h 51' 20''$, mean time, at Greenwich; and the place of the sun and Venus $2' 15' 36' 33''$; and the geocentric latitude of Venus south $9' 44''.9$. The places of the sun and Venus being computed for three hours before, and three hours after the ecliptic conjunction, the horary motion of the sun is $2' 23''.45$; of Venus retrograde $1' 33''.68$: the horary motion of Venus from the sun, therefore, $3' 57''.13$, retrograde. The horary motion of Venus in latitude is south $35''.46$. The angle of the visible way with the ecliptic $8^\circ 30' 10''$; the horary motion in that way $3' 59''.77$. The right ascension of the sun, supposing the apparent obliquity of the ecliptic $23^\circ 28' 18''$, was then $74^\circ 22' 19''.2$; and the horary motion of the sun in right ascension was $2' 34''.55$. The declination of the sun was then $22^\circ 41' 35''.9$; the horary motion in declination was $15''.33$ northwards. The angle formed by the axis of the ecliptic, and the axis of the equator, was $6^\circ 9' 34''$, decreasing hourly one minute.

The right ascension of Venus, at the ecliptic conjunction, was $74^\circ 23' 27''.2$; and the horary motion of Venus in right ascension $1' 36''.75$ retrograde. The horary motion of Venus from the sun in right ascension,

ascension was, therefore, $4' 11''.3$ retrograde. The declination of Venus was then $22^{\circ} 31' 54''.2$; and the horary motion in declination was $45''.29$, southwards: the horary motion of Venus from the sun in declination was, therefore, $1' 0''.62$, southwards.

The logarithm of the earth from the sun was then 5.006642 ; the logarithm of Venus from the sun was 4.861192 ; and the logarithm of Venus from the earth was 4.460874 . If we suppose the horizontal parallax of the sun to be $10\frac{1}{3}''$, then the horizontal parallax of Venus, as seen from the earth, will be $36''.31$; which, diminished by that of the sun, is $25''.97$. If the parallax in longitude and latitude is computed from these data, the visible horary motion of Venus from the sun in longitude will be $3' 58''.35$ retrograde, and in latitude $33''.75$ south. The longitude and latitude of the center of Venus from the sun's center, answering to the several right ascensions and declinations observed, may be determined in the following manner.

FIG. 2.

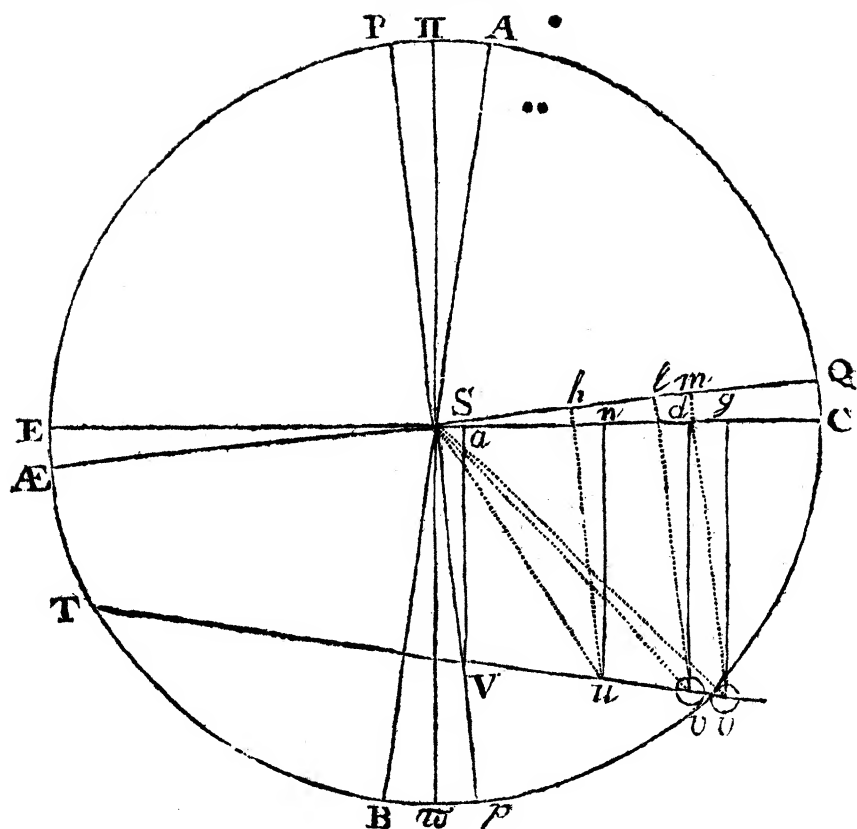


Fig. 2. Let the circle ΠE , ϖC represent the sun's disk; in which let EC be a portion of the ecliptic, $\Pi \varpi$ its axis, $\text{Æ} Q$ a parallel to the equator, $P \rho$ its axis, $TV v$ the visible path of Venus on the sun, and AB the perpendicular to that path. The angle $QSC = PS\Pi =$ the inclination of the axis of the equator to the axis of the ecliptic, is given by calculation;

ulation; then, at the internal contact, the side Sv , being the semidiameter of the sun, lessened by the semidiameter of Venus, is given, and also vl , the observed difference of declination; from whence may be found, by plain trigonometry, the angle vSl ; from which, if the angle QSC be subtracted, there will remain the angle vSd ; from whence, with Sv , may be found Sa , the difference of longitude, and vd , the difference of latitude from the sun's center. in any other position, as at u , there will be given Sh , the difference of right ascension, and uh , the difference of declination; from whence may be found the angle uSh , and the side Su : if from the angle uSh , the angle QSC be subtracted, there will remain the angle uSn ; which, with the side Su , before found, will give Sn , the difference of longitude, and un , the difference of latitude from the sun's center. At the conjunction in right ascension, SV is the observed difference of declination, and the complement of the angle QSC is $=$ the angle VSa ; from whence will be found the difference of longitude Sa , and the difference of latitude Va , from the sun's center.

1. If a mean be taken of the 4th, 5th, 6th, and 7th of Mr. Hornsby's observations, and also of the times at $17^h 56' 46''$, the right ascension of the center of Venus will be $1''.2$ before the sun's center, and the declination of it $10' 10''.6$; from whence the visible conjunction in right ascension was at $17^h 56' 31''$, and the visible declination south of the sun's center $10' 10''.4$: the visible longitude was, therefore, $1' 5''.5$ before the sun's center, and the visible latitude south of it $10' 6''.9$. From the computed

puted visible motion in longitude and latitude, by making the proper proportion, the visible ecliptic conjunction will be found at $17^h 40' 3''$ apparent time, at Shirburn, or at $17^h 44' 4''$ apparent time, at Greenwich, when the visible latitude was $9' 57''.6$ south of the sun's center. At $17^h 56' 46''$, the parallax in longitude (supposing, as above, the horizontal parallax of the sun to be $10\frac{1}{3}''$) will be $14''$, to be added to the visible longitude of Venus, to give her true longitude before the sun's center, and $20''.5$ to be subtracted from the visible latitude, to give the true latitude, as seen from the center of the earth. The true ecliptic conjunction, therefore, was at $17^h 36' 25''$ apparent time, at Shirburn, or at $17^h 40' 26''$ apparent time, at Greenwich, by making a proper proportion from the computed true motion of Venus from the sun; and the true latitude was then $9' 34.5''$ south.

2. From the mean of 10th, 11th, 12th, and 13th observations, at $19^h 24' 9''$ apparent time, at Shirburn, the observed right ascension was $5' 40''$, and the observed declination was $11' 33.6''$; from whence the visible longitude was $6' 52.2''$, and the visible latitude $10' 53''.3$, from the sun's center; and the visible ecliptic conjunction was at $17^h 40' 23''$, at Shirburn, or at $17^h 44' 24''$ apparent time, at Greenwich, with $9' 54.9''$ of visible latitude south. The parallax of longitude was $13.2''$, to be added to the visible longitude; and the parallax of latitude $18.1''$, to be subtracted from the visible latitude, to give the true latitude. The true ecliptic conjunction was, therefore, at $17^h 36' 31''$, at Shirburn, or at 17^h

17^h 40' 32'' apparent time, at Greenwich; the true latitude being then 9' 31''.6 south.

3. From the mean of the 14th and 15th observations, at 19^h 47' 29'', the observed right ascension was 7' 6''.3, and the observed declination 11' 57''.6; from whence the visible longitude was 8' 20''.5, and the visible latitude was 11' 8'', from the sun's center; and the visible ecliptic conjunction was at 17^h 41' 30'', or at 17^h 45' 31'', apparent time, at Greenwich, with visible latitude 9' 57''.2 south. The parallax of longitude was 12''.5, to be added; and the parallax of latitude 17''.4, to be subtracted, to give the true longitude and latitude. The true ecliptic conjunction was, therefore, at 17^h 37' 42'', at Shirburn, or at 17^h 41' 43'', apparent time, at Greenwich; the true latitude being then 9' 33''.9 south.

4. At the internal contact, at Shirburn, at 20^h 15' 10'', if the motion in declination, answering to 3' of time, be added to the declination observed at the 16th observation, the declination of the center of Venus from the sun's center will be 12' 24''.4; from whence the visible longitude was 10' 12''.6, and the visible latitude 11' 23'', from the sun's center; and the visible ecliptic conjunction was at 17^h 40' 57', at Shirburn, or at 17^h 44' 58', apparent time, at Greenwich, with 9' 56''.2 of visible latitude south. The parallax of longitude, to be added, was 11''.6; and the parallax of latitude 16''.5, to be subtracted, to give the true longitude and latitude. The true ecliptic conjunction was, therefore, at 17^h 37' 13'', at Shirburn, or at 17^h 41' 14'', apparent time, at Greenwich; the true latitude being 9' 33''.1 south.

5. The

5. The 2d observation made at Greenwich being dubious, if the mean of the 1st, 3d, 4th, and 5th, be taken at $19^h 48' 39''$, apparent time, at Greenwich, the observed right ascension was $6' 52''.4$, and the observed declination $11' 52''.4$; from whence the visible longitude was $8' 6''.1$, and the visible latitude $11' 4''.3$, from the sun's center. The visible ecliptic conjunction was, therefore, $17^h 46' 17''$, apparent time, at Greenwich, with $9' 55''.5$ of visible south latitude. The parallax of longitude, to be added, was $12''.6$; and the parallax of latitude, to be subtracted, $17''.3$, to give the true longitude and latitude from the sun's center. The true ecliptic conjunction, therefore, was at $17^h 42' 28''$, apparent time, at Greenwich; when the true latitude was $9' 32''.4$.

I have omitted the computation of the longitude, latitude, and of the visible and true conjunction from the internal contact, at Greenwich, and the difference of declination, as given in my last letter; because there must have been some mistake in reading the numbers of the micrometer, or in setting them, or the times, down: for they differ too much from all the above, which correspond so well with each other, (though made at different places, and with different instruments) and give the true latitude, at the ecliptic conjunction, about $8''$ less, that we cannot safely depend upon them.

If, therefore, we suppose the visible ecliptic conjunction to have happened at $17^h 45' 3''$, apparent time, at Greenwich, being the mean of the five foregoing deductions, where the greatest difference is no more than $2' 13''$ of time, or $8''$ of visible longitude,

with

with $9' 56''.3$ of visible south latitude, from the sun's center; where the greatest difference is no more than $2''.7$ in latitude, we cannot much err from the truth: and also, from the mean of the same deductions, the true ecliptic conjunction, as seen from the earth's center, will be at $17^h 41' 17''$, with $9' 33''.1$ of south latitude. The middle of the transit was, therefore, at $17^h 20' 5''$; and the nearest approach of the centers $9' 26''.8$. The latitude then was $9' 20''.6$ south; but the longitude of Venus being augmented by the aberration of light $3'' 7$, equivalent to $56''$ of time, by which the true ecliptic conjunction was accelerated, the true equated conjunction was at $17^h 42' 13''$. The error in latitude, caused by the aberration of light, was $1''.4$, by which it was diminished; the equated latitude, therefore, was $9' 34''.5$.

The equation of time was then $1' 52''$, to be subtracted from the apparent time, to give the mean; consequently, the true equated ecliptic conjunction, as seen from the earth's center, was at $17^h 40' 21''$, mean time, at Greenwich. The true place of the sun, corrected by observation, was, at that time, $2^s 15^o 36' 12''$; and, consequently, the heliocentric place of Venus was $8^s 15^o 36' 12''$, with the geocentric latitude $9' 34''.5$. Now, in this case, the geocentric latitude is to the heliocentric latitude, as the distance of Venus from the sun is to the distance of Venus from the earth; and therefore, the planet's latitude, as seen from the sun, was $3' 48''.5$. If we suppose the inclination of the orbit of Venus to be $3^o 23' 20''$, as determined by Dr. Halley and M. Cassini, the distance of Venus from the node will be

1° 4' 20''; consequently, its true place 2^d 14° 31' 52'' on the day of the transit. The effect of refraction is not taken into these calculations; because, at the first observations, when its effect would have been greatest, it amounted only to a very small part of a second.

These, my Lord, are the Conclusions, which I have been able to deduce, from the observations made at your Lordship's own observatory, and at the Royal Observatory at Greenwich. They are as faithfully related, as they were scrupulously calculated; and if they meet with the approbation of your Lordship, and of the Royal Society, I shall think myself sufficiently rewarded, for the Labour of a long and tedious calculation.

I am,

With the greatest respect,

My Lord,

Your Lordship's,

and the Royal Society's,

much obliged,

and most obedient,

humble servant,

Oxford,
Dec. 15, 1761,

Nathanael Blis.